Chapter 5

Solving Initial-Value Problem for a System of ODE's

Extended from the previous section **Solving Initial-Value Problem for ODE**, all the numerical methods are the same as described before except that they are in vector forms and the absolute value involved with the estimate of local truncation error is changed to a vector norm.

In addition to the numerical feature as the previous section, a new function of illustrating the dynamic feature of a autonomous ODE system through phase portrait is added and shown in the figure of **phase plot**.

In one of menu items, **examples,** there are several systems of ODE's Listed as examples. Among them, Lotka-Volterra, pendulum, Van der Pol, Duffing, Lorenz equations are selected to illustrate their dynamic features; linear1, linear2, linear3 are selected to illustrate the numerical phenomenon of stiff ODE systems.

Lotka-Volterra equation:

$\dot{x} = x(3 - x - 2y)$, and $\dot{y} = y(2 - x - y)$.

Pendulum equation:

$$\ddot{y} = -\sin y .$$

Van der Pol equation:

$$\ddot{y} + \mu(y^2 - 1)\dot{y} + y = 0 .$$

Duffing equation:

$$\ddot{y} + y + \varepsilon\, y^3 = 0 .$$

Lorenz equation:

$\dot{x} = \sigma(y - x)$, $\dot{y} = rx - y - xz$, and $\dot{z} = xy - bz$.

Linear 1 equation:

$\dot{x} = -10x$, and $\dot{y} = -y$.

with exact solution $x = e^{-10t}$, and $y = e^{-t}$.

Linear 2 equation:

$\dot{x} = 9x - \frac{57}{4}y$, and $\dot{y} = \frac{38}{3}x - \frac{39}{2}y$.

with exact solution

$x = \frac{1}{2}e^{-\frac{1}{2}t} + \frac{1}{2}e^{-10t}$, and $y = \frac{2}{3}e^{-10t} + \frac{1}{3}e^{-\frac{1}{2}t}$.

Linear 3 equation:

$\dot{x} = -10x$, $\dot{y} = -5y$, and $\dot{z} = -z$.

with exact solution

$x = e^{-10t}$, $y = e^{-5t}$, and $z = e^{-t}$.

References:

【1】 R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, Boston, 1993.

【2】 S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Addison-Wesley, New York, 1994.