

# Chapter 4 Numerical Differentiation and Integration

## Riemann Integral

$$f \in C[a, b], \quad x_{k-1} \leq x_k^* \leq x_k.$$

$$R = \sum_{k=1}^n f(x_k^*) \Delta x_k, \quad \text{where } \Delta x_k = x_k - x_{k-1}, \quad k = 1, 2, \dots, n.$$

If  $x_k^* = x_k$ ,  $R$  is called right Riemann sum.

If  $x_k^* = x_{k-1}$ ,  $R$  is called left Riemann sum.

The definite integral of  $f$  over  $[a, b]$  is defined as

$$\int_a^b f(x) dx = \lim_{\max \Delta x_k \rightarrow 0} \sum_{k=1}^n f(x_k^*) \Delta x_k.$$

References:

- 【1】 L. I. Holder, J. DeFranza and J. M. Pasachoff, *Calculus*, Brooks/Cole, Pacific Grove, 1998.