Chapter 4　Numerical Differentiation and Integration

Elementary Numerical Integration

1. Closed Newton-Cotes formulas：

The following $\sum_{i=0}^{n} a_i f(x_i)$ denotes the $(n+1)$-points closed Newton-Cotes formula with $x_0 = a, x_n = b,$ and $h = (b-a)/n$.

n=1：Trapezoidal rule

$$\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}[f(x_0) + f(x_1)] - \frac{h^3}{12} f''(\xi),$$

n=2：Simpson's rule

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi),$$

n=3：Simpson's three-eighths rule

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)] - \frac{3h^5}{80} f^{(4)}(\xi),$$

n=4：

$$\int_{x_0}^{x_4} f(x)dx = \frac{2h}{45}[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)] - \frac{8h^7}{945} f^{(6)}(\xi).$$

2. Open Newton-Cotes formulas：

The following $\sum_{i=0}^{n} a_i f(x_i)$ denotes the $(n+1)$-points open Newton-Cotes formula with $x_{-1} = a, x_{n+1} = b$, and $h = (b-a)/(n+2)$.

n=0：Midpoint rule

$$\int_{x_{-1}}^{x_1} f(x)dx = 2hf(x_0) + \frac{h^3}{3} f''(\xi),$$

n=1：

$$\int_{x_{-1}}^{x_2} f(x)dx = \frac{3h}{2}[f(x_0) + f(x_1)] + \frac{3h^3}{4} f''(\xi),$$

n=2：

$$\int_{x_{-1}}^{x_3} f(x)dx = \frac{4h}{3}[2f(x_0) - f(x_1) + 2f(x_2)] + \frac{14h^5}{45} f^{(4)}(\xi),$$

n=3：

$$\int_{x_{-1}}^{x_4} f(x)dx = \frac{5h}{24}[11f(x_0) + f(x_1) + f(x_2) + 11f(x_3)] + \frac{95h^5}{144} f^{(4)}(\xi).$$

References:

【1】 R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, Boston, 1993.