

Chapter 4 Numerical Differentiation and Integration

Gaussian Quadrature

$$\int_{-1}^1 f(x)dx \approx \sum_{i=1}^n c_i f(x_i) , \quad c_i = \int_{-1}^1 \prod_{j=1, j \neq i}^n \frac{x - x_j}{x_i - x_j} dx ,$$

where $\{x_i\}_{i=1}^n$ are the roots of the Legendre polynomial of degree n .

$\sum_{i=1}^n c_i f(x_i)$ approximates $\int_{-1}^1 f(x)dx$ in infinite order of accuracy as

$n \rightarrow \infty$, that is, faster than any algebraic order.

References:

- 【1】** R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, Boston, 1993.