Chapter 4 Numerical Differentiation and Integration

Composite Numerical Integration

1. Composite Trapezoidal rule：

   Let $f \in C^2[a,b], h = (b-a)/n,$ and $x_j = a + jh$ for each $j = 0,1,\ldots,n.$

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)] - \frac{b-a}{12}h^2 f''(\xi).$$

2. Composite Simpson's rule：

   Let $f \in C^4[a,b], n$ be even, $h = (b-a)/n$, and $x_j = a + jh$ for each $j = 0,1,\ldots,n.$

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{b-a}{180}h^4 f^{(4)}(\xi).$$

3. Composite Midpoint rule：

   Let $f \in C^2[a,b],\ h = (b-a)/n$, and $x_j = a + jh$ for each $j = 0,\ldots,n.$

$$\int_a^b f(x)dx = h\sum_{j=1}^{n} \frac{f(x_{j-1}) + f(x_j)}{2} + \frac{b-a}{24}h^2 f''(\xi).$$

References:

【1】 R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, Boston, 1993.