Chapter 4 Numerical Differentiation and Integration

Instability of Numerical Differentiation by Finite Approximation

Approximating $f'(x_0)$:

1. Forward difference

$$f'(x_0)=\frac{f(x_0+h)-f(x_0)}{h}-\frac{h}{2}f''(\xi).$$

2. Central difference（3 points）

$$f'(x_0)=\frac{f(x_0+h)-f(x_0-h)}{2h}-\frac{h^2}{6}f'''(\xi).$$

3. Central difference（5 points）

$$f'(x_0)=\frac{f(x_0-2h)-8f(x_0-h)+8f(x_0+h)-f(x_0+2h)}{12h}+\frac{h^4}{30}f^{(5)}(\xi).$$

Approximating $f''(x_0)$:

1. Central difference（3 points）

$$f''(x_0)=\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}-\frac{h^2}{12}f^{(4)}(\xi).$$

2. Central difference（5 points）

$$f''(x_0)=\frac{-f(x_0+2h)+16f(x_0+h)-30f(x_0)+16f(x_0-h)-f(x_0-2h)}{12h^2}$$

$$+\frac{h^4}{90}f^{(6)}(\xi).$$

All numerical differentiation methods by finite difference approximation are unstable due to the growth of roundoff error as $h \to 0$.

References:

【1】 R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, Boston, 1993.