

Chapter 2 Solutions of Equations in One Variable

Logistic map

$$x_{n+1} = rx_n(1 - x_n)$$

x_0 : initial value, $0 \leq x_0 \leq 1$.

r : parameter, $1 \leq r \leq 4$.

$1 < r < 3$, x_n converges to a fixed point.

$3 \leq r < 3.449\dots$, period 2.

$3.449\dots \leq r < 3.54409\dots$, period 4.

$3.54409\dots \leq r < 3.5644\dots$, period 8.

$3.5644\dots \leq r < 3.568759\dots$, period 16.

⋮

$3.569946\dots \leq r$, period ∞ .

Logistic mapping shows a route to chaos by period-doublings.

References:

- 【1】 S. H. Strogatz, *Nonlinear Dynamics and Chaos*, Addison-Wesley, New York, 1994.