# Chapter 6 Numerical Linear Algebra

## Norm of Vectors and Matrices

The $l_1, l_2$, and $l_\infty$ norms of a vector $\mathbf{x} = (x_1, x_2, \ldots, x^n)^t$ are defined by

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|, \quad \|\mathbf{x}\|_2 = \left\{ \sum_{i=1}^{n} x_i^2 \right\}^{1/2}, \quad \text{and } \|\mathbf{x}\|_\infty = \max_{1 \le i \le n} |x_i|.$$

If $\|\cdot\|$ is any vector norm on $R^n$, then $\|\mathbf{A}\| = \max_{\|\mathbf{x}\|=1} \|\mathbf{Ax}\|$ is the definition for a matrix norm. For example, the $l_2$ norm of a matrix $\mathbf{A}$ is defined by $\|\mathbf{A}\|_2 = \max_{\|\mathbf{x}\|_2=1} \|\mathbf{Ax}\|_2$. Furthermore, it can be shown that the $l_1, l_2$, and $l_\infty$ norms of a $n \times n$ matrix $\mathbf{A}$ can be computed by:

$$\|\mathbf{A}\|_1 = \max_{1 \le j \le n} \sum_{j=1}^{n} |a_{ij}|, \quad \|\mathbf{A}\|_2 = \sqrt{\rho(\mathbf{A}^t\mathbf{A})}, \quad \text{and}$$

$\|\mathbf{A}\|_\infty = \max_{1 \le i \le n} \sum_{j=1}^{n} |a_{ij}|$, where $\rho(\cdot)$ is the spectral radius.

Motivated by Moler's eigen show, in addition to the eigenvalue and eigenvector, this software shows the geometric meaning of $l_1, l_2$, and $l_\infty$ norms of a matrix according to their definitions.

References:

【1】 R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, Boston, 1993.